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TWO-PHASE BOUNDARY-LAYER TREATMENT OF FREE-CONVECTION FILM BOILING

KANEYASU NISHIKAWA* and TAKEHIRO ITO†
Department of Mechanical Engineering, Kyushu University,
Hakozaki, Fukuoka-shi, Japan

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Abstract—With the well-known boundary-layer theory as a basis, a theoretical study has been made of film-boiling heat transfer from an isothermal vertical plate to a subcooled stagnant liquid. The results are also applicable to film-boiling heat transfer from an isothermal horizontal cylinder.

By assuming a thin stable vapor film, the boundary-layer equations were transformed into a system of ordinary differential equations by the similarity relations, and the heat-transfer characteristics were found to be correlated by six parameters. The formulation of the two-phase flow problem in this paper differed from that by Sparrow and Cess, only in that nonslip conditions of interfacial velocity and shearing stress were required, instead of the zero interfacial-velocity condition used by them.

We first obtained solutions for some arbitrarily chosen sets of parameters to see how each parameter affected the heat-transfer rate, this proved Sparrow and Cess's evaluation of the effects of subcooling on heat-transfer coefficient to be wrong. Then we found the heat-transfer characteristics of water at 1 atm.

Differential equations were solved on electronic digital computers by successive approximation.

NOMENCLATURE

c_p, specific heat;
d, diameter of horizontal cylinder;
f, dimensionless velocity function, equations (10) and (34);
g, acceleration due to gravity;
h, height of vertical plate;
l, latent heat;
m, dimensionless constant, equation (12);
q, heat flux on heating surface;
r, radius of horizontal cylinder;
u, velocity component in x-direction;
v, velocity component in y-direction;
w, rate of mass flow;
x, y, co-ordinates;
A, a dimensionless quantity, equation (17′);
B, a dimensionless quantity, equation (18′);
C, a dimensionless quantity, equation (19′);
D, a dimensionless quantity, equation (20′);
Gr, Grashof number, equation (29);
Gr_d, Grashof number in case of horizonta cylinder, equation (41);
M, constant, equation (37);
N, constant, equation (38);
Nu, local Nusselt number, equation (29);
$\overline{Nu}$, average Nusselt number, equation (30);
$\overline{Nu}_d$, average Nusselt number in case of horizontal cylinder, equation (41);
Pr, Prandtl number;
R, $\rho\mu$ ratio, equation (24);
Sc, dimensionless degree of subcooling, equation (26);
Sp, dimensionless degree of superheating, equation (20′);
T, temperature;
ΔT_L, degree of subcooling;
ΔT_V, degree of superheating;
X, angle measured from bottom of cylinder, x/r.

Greek symbols
β, coefficient of cubic expansion;
γ, function of X, equation (40);
δ, thickness of vapor film;

* Professor of Mechanical Engineering, Kyushu University, Fukuoka, Japan
† Student of Graduate School of Kyushu University.

η, similarity variable, equations (9) and (33);
θ, dimensionless temperature, equations (11), (35);
λ, thermal conductivity;
μ, absolute viscosity;
ν, kinematic viscosity;
ρ, density;
ϕ, function of X, equation (40);
Φ, stream function, equations (10) and (34).

Suffixes

V, vapor;
L, liquid;
w, heating surface;
s, saturated condition;
i, the vapor–liquid interface;
∞, the values far enough from heating surface.

1. INTRODUCTION

IT WAS Bromley [1] who first treated film-boiling heat transfer from a purely analytical viewpoint. He developed his theory on the assumption that the temperature distribution within a vapor film is linear, omitting the inertia term of the equation of motion concerning the vapor film as in Nusselt's water-film theory [2] of film condensation, and obtained a certain degree of success in the case of saturated film boiling. Later on, attempts were made to improve the theory in detail, but none of them proved epoch-making. In recent years, however, a great improvement has been made over the old theories by the development of a new mode of interpreting the phenomena of film boiling, viz. the so-called conception of the two-phase boundary layer. The treatment according to the two-phase boundary-layer theory is a method of solving the fundamental differential equations that hold for the vapor film and liquid film respectively by the application of the boundary-layer treatment both to the vapor film that grows along the heating surface and to the liquid film where upward movement has been induced by the ascent of vapor. Because of their formidable complexity we often turned to the aid of an electronic computer in the integration of the fundamental differential equations.† It seems that this theory has attracted general attention, probably receiving impetus from the sucess of the treatment due to similar ideas in film condensation. Koh [3] made a sound treatment of the saturated film boiling of free-convection from a vertical plate according to the two-phase boundary-layer theory, and later on, Sparrow and Cess [4] have carried out an analysis on surface film boiling from a vertical plate by modifying the solution without a degree of subcooling. In the latter analysis, however, the bold assumption has been made of neglecting the velocity component in the tangential direction at the vapor–liquid interface, probably due to the difficulty of getting a solution.

In the present paper, the authors aim to clarify the character of free-convection film boiling in the case when the temperature of liquid is lower than saturation temperature, by obtaining solutions for the problem without using the assumption in which the tangential component of velocity at the vapor–liquid interface is neglected.

† The authors depended mainly on an IBM 7090 computer and supplementarily on OKITAC 5090 A and H computers.

2. ANALYSIS

2.1. *Physical model*

As shown in Fig. 1, the analysis is concerned with the case where a solid wall is set vertically in the wide space of liquid of temperature T_∞ which

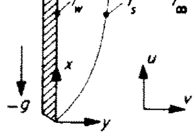


FIG. 1. Physical model and co-ordinates for vertical plate.

is lower than the saturation temperature T_s by the degree of subcooling ΔT_L. The uniform wall temperature T_w is higher than the saturation temperature T_s by the degree of superheating ΔT_V. It will be assumed that the physical properties of vapor and liquid do not change with temperature, and furthermore, that there exists a smooth vapor–liquid interface and laminar boundary layer and that radiative heat transfer is left out of consideration.

2.2. *Fundamental equations*

Assuming that the vapor film forms itself into a boundary layer and applying the fundamental equations of heat transfer from the viewpoint of a boundary layer, we obtain the following three equations:

$$\frac{\partial u_V}{\partial x} + \frac{\partial v_V}{\partial y} = 0 \tag{1}$$

$$u_V \frac{\partial u_V}{\partial x} + v_V \frac{\partial u_V}{\partial y} = \frac{g(\rho_L - \rho_V)}{\rho_V} + \nu_V \frac{\partial^2 u_V}{\partial y^2} \tag{2}$$

$$u_V \frac{\partial T_V}{\partial x} + v_V \frac{\partial T_V}{\partial y} = \frac{\lambda_V}{g \rho_V c_{pV}} \frac{\partial^2 T_V}{\partial y^2} \tag{3}$$

What is different from the case of single-phase free-convection is that the first term of the right-hand side of equation (2) can be expressed as a constant.

Now, the flow of the vapor phase will induce an upward movement of the liquid phase through the vapor–liquid interface. If similar equations to the above-mentioned three equations are written on the assumption that the liquid close to this vapor–liquid interface has also a characteristic of something like a boundary layer, the result will be as follows:

$$\frac{\partial u_L}{\partial x} + \frac{\partial v_L}{\partial y} = 0 \tag{4}$$

$$u_L \frac{\partial u_L}{\partial x} + v_L \frac{\partial u_L}{\partial y} = g\beta_L(T_L - T_\infty) + \nu_L \frac{\partial^2 u_L}{\partial y^2} \tag{5}$$

$$u_L \frac{\partial T_L}{\partial x} + v_L \frac{\partial T_L}{\partial y} = \frac{\lambda_L}{g \rho_L c_{pL}} \frac{\partial^2 T_L}{\partial y^2} \tag{6}$$

2.3. *Similarity transformation*

Here we have to obtain a solution for the six equations given above under the boundary conditions and the interface matching conditions at the vapor–liquid interface (to be explained later); there are two methods of solving those equations that are frequently used. The one is the similarity transformation proposed by Blasius-Pohlhausen and the other the profile method depending upon the integrated boundary-layer equations advocated by von Kármán. The authors have adopted the former method since they thought it difficult to assume the profiles of temperature and velocity properly.

Therefore, the following stream functions will be introduced first:

$$u_V = \frac{\partial \Phi_V}{\partial y}, \quad v_V = -\frac{\partial \Phi_V}{\partial x} \tag{7}$$

$$u_L = \frac{\partial \Phi_L}{\partial y}, \quad v_L = -\frac{\partial \Phi_L}{\partial x} \tag{8}$$

Next, the so-called similarity variables will be defined as follows, including unknown constants m_V, m_L:

$$\eta_V = m_V x^{-\frac{1}{4}} y, \quad \eta_L = m_L x^{-\frac{1}{4}} y. \tag{9}$$

It will be further assumed that the stream functions and temperature can be expressed non-dimensionally as functions solely of similarity variables as follows:

$$f_V(\eta_V) = \frac{\Phi_V}{4 m_V \nu_V x^{\frac{3}{4}}}, \quad f_L(\eta_L) = \frac{\Phi_L}{4 m_L \nu_L x^{\frac{3}{4}}} \tag{10}$$

$$\theta_V(\eta_V) = \frac{T_V - T_s}{T_w - T_s}, \quad \theta_L(\eta_L) = \frac{T_L - T_\infty}{T_s - T_\infty} \tag{11}$$

Now, four differential equations are to be obtained with respect to the unknown functions f, θ by substituting these relations into the fundamental equations (1)–(6). If the constants m_V, m_L are determined such that the four differential equations do not include x, the result will be as follows:

$$m_V = \left[\frac{g(\rho_L - \rho_V)}{4\nu_V^2 \rho_V}\right]^{\frac{1}{4}}, \quad m_L = \left[\frac{g\beta_L(T_s - T_\infty)}{4\nu_L^2}\right]^{\frac{1}{4}} \tag{12}$$

The fundamental equations (1)–(6) can be transformed into the following four ordinary differential equations by the above-mentioned similarity transformation.

$$f_V''' + 3 f_V f_V'' - 2(f_V')^2 + 1 = 0 \tag{13}$$

$$\theta_V'' + 3 Pr_V f_V \theta_V' = 0 \tag{14}$$

$$f_L''' + 3 f_L f_L'' - 2(f_L')^2 + \theta_L = 0 \tag{15}$$

$$\theta_L'' + 3 Pr_L f_L \theta_L' = 0 \tag{16}$$

where the primes represent differentiation with repect to η_V or η_L in accordance with suffixes V or L.

2.4. *Boundary conditions and matching conditions* [5]

In the theory of the two-phase boundary layer, the main thing is to describe clearly the characteristics of continuity and matching of the various physical quantities in the boundary surface of the two phases, and at this point each method has come to show its characteristics.

First of all, it is plain that the mass transfer crossing the vapor–liquid interface is continuous; this can be expressed as follows and from it the first condition is obtained:

$$\rho_V\left(v_V - u_V\frac{\mathrm{d}\delta}{\mathrm{d}x}\right)_i = \rho_L\left(v_L - u_L\frac{\mathrm{d}\delta}{\mathrm{d}x}\right)_i \quad (17)$$

$$(f_L)_i = A\cdot(f_V)_i,$$

$$A \equiv (\beta_L\Delta T_L)^{-\frac{1}{4}}\left(\frac{\mu_V}{\mu_L}\right)^{\frac{1}{2}}\left(\frac{\rho_V}{\rho_L}\right)^{\frac{1}{4}} \quad (17')$$

The second conditional equation is obtained by assuming the velocity component in tangential direction at the vapor–liquid interface to have no slip; this will be expressed approximately as follows in the concept of boundary layer:

$$(u_V)_i = (u_L)_i \quad (18)$$

$$(f_L')_i = B\cdot(f_V')_i,\quad B \equiv (\beta_L\Delta T_L)^{-\frac{1}{2}}\left(\frac{\rho_V}{\rho_L}\right)^{-\frac{1}{2}} \quad (18')$$

For the third condition it will be assumed that the same extreme values of the shearing stress that acts in a tangential direction of the vapor–liquid interface arise from both the vapor and the liquid phases. As an expression in the concept of the boundary layer, there will be obtained the following equations:

$$\mu_V\left(\frac{\partial u_V}{\partial y}\right)_i = \mu_L\left(\frac{\partial \mu_L}{\partial y}\right)_i \quad (19)$$

$$(f_L'')_i = C\cdot(f_V'')_i,\quad C \equiv A\cdot B \quad (19')$$

According to the Sparrow and Cess analysis, the conditions of equations (18) and (19) are replaced by $(u_V)_i = (u_L)_i = 0$.

On the other hand, the heat balance at the vapor–liquid interface can be expressed as follows:

$$\left.\begin{aligned}\left(-\lambda_V\frac{\partial T_V}{\partial y}\right)_i &= -g(w)_i l + \left(-\lambda_L\frac{\partial T_L}{\partial y}\right)_i\\ (w)_i &= \rho_V\left(v_V - u_V\frac{\mathrm{d}\delta}{\mathrm{d}x}\right)\end{aligned}\right\} \quad (20)$$

$$\left.\begin{aligned}Sp &= -3\frac{(f_V)_i}{(\theta_V')_i} + D\frac{(\theta_L')_i}{(\theta_V')_i}\\ Sp &\equiv \frac{c_{pV}\Delta T_V}{lPr_V}\\ D &\equiv \left(\frac{c_{pV}}{lPr_V\beta_L}\right)(\beta_L\Delta T_L)^{\frac{5}{4}}\left(\frac{\mu_V}{\mu_L}\right)^{\frac{1}{2}}\left(\frac{\lambda_V}{\lambda_L}\right)^{-1}\left(\frac{\rho_V}{\rho_L}\right)^{-\frac{1}{4}}\end{aligned}\right\} \quad (20')$$

Other boundary conditions will be written almost the same as for the case of a single phase as follows:

$$y = 0:\quad u_V = v_V = 0,\quad T_V = T_w \quad (21)$$

$$\eta_V = 0:\quad f_V = f_V' = 0,\quad \theta_V = 1 \quad (21')$$

$$y = \delta:\quad T_V = T_L = T_s \quad (22)$$

$$(\eta_V)_i = (\eta_L)_i = \eta_i:\quad \theta_V = 0,\quad \theta_L = 1 \quad (22')$$

$$y \to \infty:\quad u_L \to 0,\quad T_L \to T_\infty \quad (23)$$

$$\eta_L \to \infty:\quad f_L' \to 0,\quad \theta_L \to 0 \quad (23')$$

Parameters A, B, C and D in the foregoing considerations will be expressed by the $\rho\mu$ ratio, as frequently used in the theory of the two-phase boundary layer:

$$R \equiv \sqrt{\left(\frac{(\rho\mu)_V}{(\rho\mu)_L}\right)} \quad (24)$$

and the dimensionless number V used in the analysis of Sparrow and Cess [4] (which is defined as R in their paper)

$$V \equiv R\left(\frac{\rho_V}{\rho_L}\right)^{-\frac{1}{4}}\left(\frac{c_{pL}}{\beta_L l}\right)^{\frac{1}{4}} \quad (25)$$

and the degree of subcooling Sc, which is rendered dimensionless

$$Sc \equiv \frac{c_{pL}\Delta T_L}{l} \quad (26)$$

as well as by the Prandtl number of the liquid Pr_L as follows:

$$A \equiv Sc^{-\frac{1}{4}}V, \quad B \equiv R^{-2}\, Sc^{-\frac{1}{2}}V^2, \quad C \equiv AB, \quad D \equiv Pr_L^{-1}\, Sc^{\frac{3}{4}}V^{-1} \tag{27}$$

The parameters required to specify the situation of the system will be the following six:

$$Pr_V, Pr_L, R, V, Sc, Sp.$$

This is one more than the case of Sparrow and Cess [4] and there occurs the necessity to designate R additionally for the matching conditions of equations (18) and (19).

2.5. *Heat transfer*

If the heat flux q is evaluated by transformed quantities, the result is as follows:

$$q = -\lambda_V \left(\frac{\partial T_V}{\partial y}\right)_w = \lambda_V \Delta T_V m_V x^{-\frac{1}{4}}(-\theta_V')_w \tag{28}$$

Letting the representative temperature difference be the superheat ΔT_V and representing thermal conductivity by its value on the vapor side λ_V, we obtain the following expression for the Nusselt number:

$$\left.\begin{aligned} &\frac{Nu}{(Gr/4)^{\frac{1}{4}}} = -(\theta_V')_w \\ &Nu \equiv \frac{qx}{\lambda_V \Delta T_V}, \quad Gr \equiv \frac{g(\rho_L - \rho_V)x^3}{\nu_V^2 \rho_V} \end{aligned}\right\} \tag{29}$$

The average Nusselt number for the total heat transfer from the lowest end of the heating surface up to a height h will be as follows:

$$\left.\begin{aligned} &\frac{3}{4}\frac{\overline{Nu}}{(Gr/4)^{\frac{1}{4}}} = -(\theta_V')_w \\ &\overline{Nu} \equiv \frac{\bar{q}h}{\lambda_V \Delta T_V}, \quad \bar{q} \equiv \frac{1}{h}\int_0^h q\, \mathrm{d}x \end{aligned}\right\} \tag{30}$$

2.6. *The horizontal cylinder*

The case to be considered here is that of a horizontal cylinder as shown in Fig. 2 instead of one whose heating surface is a vertical plate. When the fundamental equations are written for both vapor and liquid phases under the same assumptions as in the previous case, the fundamental equations (1)–(6) are found to hold as they are only altered by multiplying the first terms of the right-hand side of equations (2) and (5) respectively by sin (x/r).

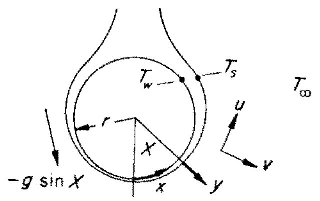


FIG. 2. Physical model and co-ordinates for horizontal cylinder.

Next, stream functions and similarity transformations for these fundamental equations will be determined as below:

$$u_V = \frac{\partial \Phi_V}{\partial y}, \quad v_V = -\frac{\partial \Phi_V}{\partial x} \tag{31}$$

$$u_L = \frac{\partial \Phi_L}{\partial y}, \quad v_L = -\frac{\partial \Phi_L}{\partial x} \tag{32}$$

$$\eta_V = \frac{N_V \gamma(X) y}{r}, \quad \eta_L = \frac{N_L \gamma(X) y}{r} \tag{33}$$

$$f_V(\eta_V) = \frac{\Phi_V}{M_V \phi(X)}, \quad f_L(\eta_L) = \frac{\Phi_L}{M_L \phi(X)} \tag{34}$$

$$\theta_V(\eta_V) = \frac{T_V - T_s}{T_w - T_s}, \quad \theta_L(\eta_L) = \frac{T_L - T_\infty}{T_s - T_\infty} \tag{35}$$

$$X = \frac{x}{r} \tag{36}$$

Furthermore, constants M and N will be chosen appropriately to obtain the following:

$$M_V \equiv \nu_V \left[\frac{g(\rho_L - \rho_V)r^3}{\nu_V^2 \rho_V}\right]^{\frac{1}{4}}, \quad M_L \equiv \nu_L \left[\frac{g\beta_L \Delta T_L r^3}{\nu_L^2}\right]^{\frac{1}{4}} \tag{37}$$

$$N_V \equiv \left[\frac{g(\rho_L - \rho_V)r^3}{\nu_V^2 \rho_V}\right]^{\frac{1}{4}}, \quad N_L \equiv \left[\frac{g\beta_L \Delta T_L r^3}{\nu_L^2}\right]^{\frac{1}{4}} \tag{38}$$

On the other hand, ϕ and γ will be assumed to satisfy the following relations as functions of X:

$$\left.\begin{aligned} &\phi\gamma^3 = \sin X \\ &\phi\gamma^3 \frac{\mathrm{d}\phi}{\mathrm{d}X} + \phi^2\gamma \frac{\mathrm{d}\gamma}{\mathrm{d}X} = 2 \sin X \\ &\frac{1}{\gamma}\frac{\mathrm{d}\phi}{\mathrm{d}X} = 3 \end{aligned}\right\} \quad (39)$$

For the functions ϕ and γ that approximately satisfy equations (39) together with the appropriate boundary conditions, Yamagata [6] has given the following equations:

$$\left.\begin{aligned} &\phi = \left[4 \int_0^x \sin^{\frac{1}{3}} X \,\mathrm{d}X \right]^{\frac{3}{4}} \\ &\gamma = \frac{\sin^{\frac{1}{3}} X}{\left[4 \int_0^x \sin^{\frac{1}{3}} X \,\mathrm{d}X\right]} \end{aligned}\right\} \quad (40)$$

(For numerical values, see the appendix).

Through the introduction of the foregoing similarity transformations, together with ϕ and γ, equations (13)–(23) in the case of a vertical plate hold formally in this case too, but the expression for the Nusselt number should be altered as follows:

$$\left.\begin{aligned} &\frac{1}{2^{\frac{1}{4}}\bar{\gamma}} \frac{\overline{Nu}_d}{(Gr_d)^{\frac{1}{4}}} = -(\theta'_V)_w \\ &\overline{Nu}_d \equiv \frac{d \cdot \int_0^{\pi} q \,\mathrm{d}X}{\pi \lambda_V \Delta T_V} \\ &Gr_d \equiv \frac{g(\rho_L - \rho_V)d^3}{\nu_V^2 \rho_V} \\ &2^{\frac{1}{4}}\bar{\gamma} = 2^{\frac{1}{4}} \cdot \frac{1}{\pi} \int_0^{\pi} \gamma \,\mathrm{d}X = 0{\cdot}728 \end{aligned}\right\} \quad (41)$$

2.7. *Numerical calculation*

In order to apply successive approximation in the actual numerical calculation, equations (13) and (15) will be linearized as follows:

$$f_V''' + 3\bar{f}_V f_V'' - 2\bar{f}_V' f_V' + 1 = 0 \quad (13')$$

$$f_L''' + 3\bar{f}_L f_L - 2\bar{f}_L' f_L + \bar{\theta}_L = 0 \quad (15')$$

provided that $\bar{f}_V'$, $\bar{f}_V$, $\bar{f}_L$, $\bar{f}_L'$ and $\bar{\theta}_L$ are the values obtained in the latest trial in the process of successive approximation. These are the second-order ordinary differential equations about f_V' and f_L', but if each is replaced by an approximate finite difference equation, they will reduce to simultaneous linear algebraic equations.

When such preparation has been made, a numerical solution will be sought by successive approximation, as explained below. First, in advance of the calculation, the five parameters Pr_V, Pr_L, R, V and Sc, that is, the six parameters defined in 3.4 other than the degree of dimensionless superheating Sp, will be designated together with $(\eta_V)_i$ of dimensionless vapor film thickness. Then assuming the estimated values of $\bar{f}_V'$, $\bar{f}_L'$ as the zeroth approximation, we can evaluate $\bar{f}_V$ and $\bar{f}_L$ at once by an appropriate numerical integration. If $\bar{f}_L$ is already known, equation (16) can be integrated with respect to $\bar{\theta}_L$ as follows:

$$\bar{\theta}_L = 1 - \frac{\int_{(\eta_L)_i}^{\eta_L} \exp\left(-3Pr_L \int_{(\eta_L)_i}^{\eta_L} \bar{f}_L \,\mathrm{d}\eta_L\right) \mathrm{d}\eta_L}{\int_{(\eta_L)_i}^{\infty} \exp\left(-3Pr_L \int_{(\eta_L)_i}^{\eta_L} \bar{f}_L \,\mathrm{d}\eta_L\right) \mathrm{d}\eta_L} \quad (16')$$

Therefore, we can easily calculate $\bar{\theta}_L$ from $\bar{f}_L$ in question.

Now that a set of $\bar{f}_V$, $\bar{f}_V'$, $\bar{f}_L$, $\bar{f}_L'$ and $\bar{\theta}_L$ has been made ready in this way, new f_V' and f_L' will be evaluated by solving the simultaneous linear algebraic equations mentioned above [7].

Furthermore, by the repetition of a similar process, max $(|f_V' - \bar{f}_V'|)$ and max $(|f_L' - \bar{f}_L'|)$ will be both rendered small enough. Then f_V, f_L and θ_L are also found to converge. On the other hand, θ_V can also be evaluated easily from the value of f_V and the integration of equation (14) similar to the equation (16′). Lastly, it is possible to determine the dimensionless superheat Sp by equation (20′) and the characteristics of heat transfer by equations (29), (30) and (41).

Fundamentally, all we have to do for correlating heat transfer is to apply such a numerical calculation as mentioned above to the combined domain which is important for practical use of the six parameters. Considering that the number of parameters is exceedingly great and that the domain in demand for practice is not always clear, the authors carried out calculations first with respect to some imaginary combinations of the parameters and then analyzed heat transfer for water at atmospheric pressure.

3. RESULTS AND CONSIDERATION

3.1. *Velocity and temperature distribution*

What characterizes the present analysis is that, instead of adopting Sparrow and Cess's assumption to neglect the velocity component in a tangential direction at the vapor–liquid interface, the authors used as matching conditions such assumptions as having no slip in the velocity component and that the extreme values from both vapor and liquid phases of the shearing forces acting in a tangential direction to the interface are equal.

Figure 3 shows the velocity distributions in the case of $Pr_V = 1{\cdot}0$, $Pr_L = 2{\cdot}0$, $R = 0{\cdot}1$, $V = Sc = 0{\cdot}05$, and there is sometimes observed, according to Sp or the thickness of vapor film, the maximum velocity at the vapor–liquid interface, with an implication that it is quite unreasonable to put zero for the velocity at the interface. Considering, on the other hand, the impropriety of treating an exceedingly thick vapor film, though the velocity at the vapor–liquid interface approaches to zero in this case, it seems to us inconceivable to admit Sparrow and Cess's assumption, even from the viewpoint of velocity distribution only.

Figure 4 shows the temperature distributions corresponding to the velocity distributions mentioned above, where it is observed that such linearity of temperature distribution as assumed by Bromley with the thickening of the vapor film is lost.

3.2. *Comparison with the former solution*

In Fig. 5, the solutions proposed in this paper are compared with the former ones, in which only two sets of parameters, i.e. $Pr_V = 1{\cdot}0, Pr_L = 2{\cdot}0$, $R = 0{\cdot}1$, $V = Sc = 0{\cdot}005$ and $Pr_V = Pr_L = 1{\cdot}0$, $R = 0{\cdot}1$, $V = Sc = 0{\cdot}005$ are adopted, the object being to make a comparison. For abscissa the dimensionless degree of superheating Sp is taken, and for ordinate $Nu/(Gr/4)^{\frac{1}{4}}$. The thickness of vapor film there is, therefore, increased with the increase of superheating degree on a single curve. In the limit where the vapor film becomes thick enough, $Nu/(Gr/4)^{\frac{1}{4}}$ is considered to approach gradually to 0·567 given by Ostrach's solution for a single phase for Prandtl number which is equivalent to that on the vapor side (1·0 in the present case).

Now the solutions obtained in the present paper are designated Nu_A and Nu_B in the figure in which parameters are all selected as the same

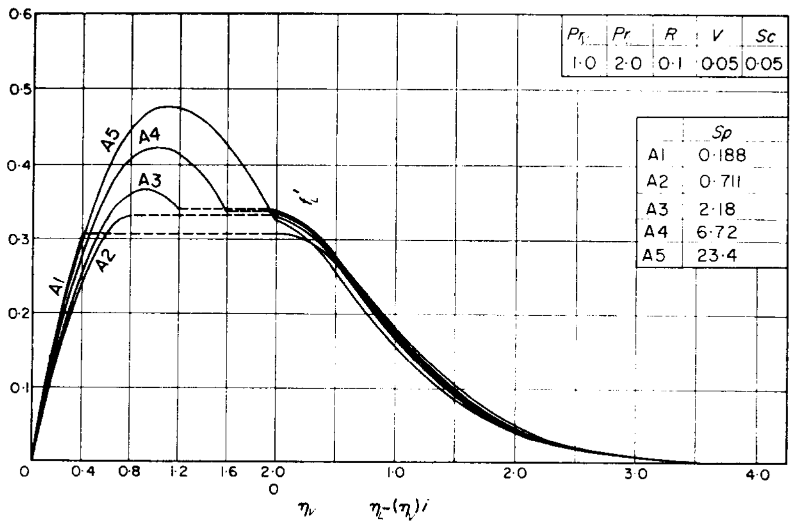


FIG. 3. Examples of velocity distribution.

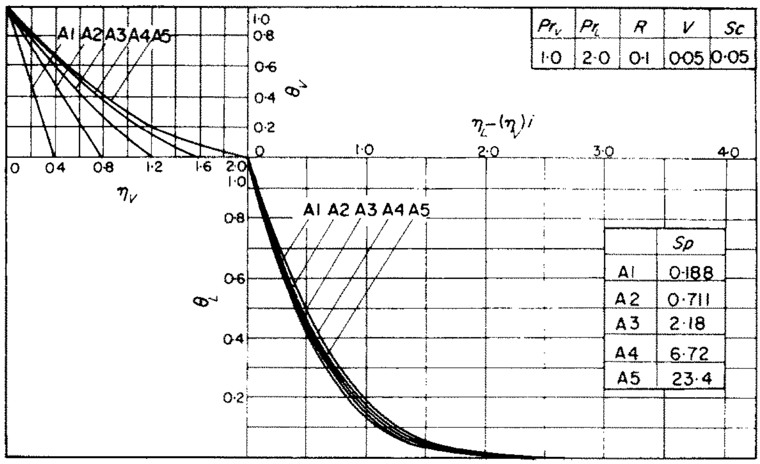


FIG. 4. Examples of temperature distribution.

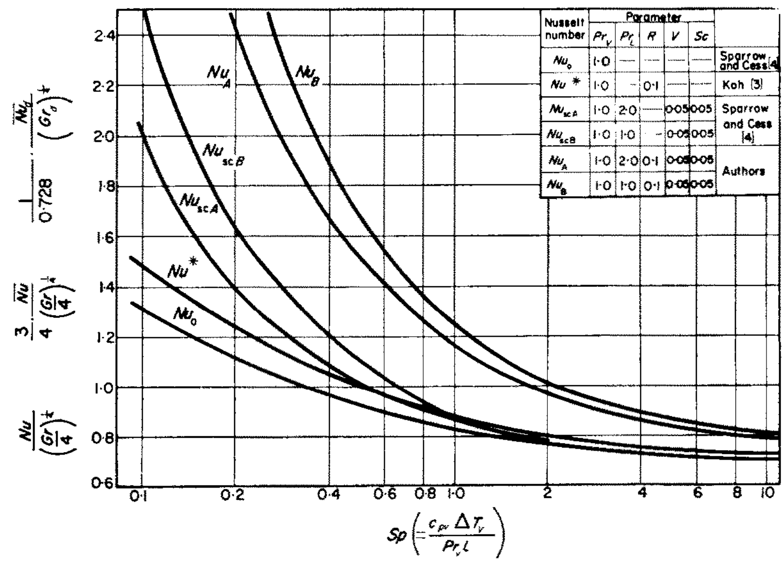


FIG. 5. Comparison of heat transfer result with the former solution.

as in the case of Koh's solution [3] for saturated film boiling Nu^* (the matching conditions at the vapor–liquid interface are the same as that in the present paper), and for the case of Sparrow and Cess's solutions [4] of surface film boiling $Nu_{sc,A}$ and $Nu_{sc,B}$ are used (the solution in which velocity component at the interface is neglected) corresponding to Nu_A and Nu_B respectively. In the case where the degree of subcooling is large relative to the degree of superheating (the part on the left side of the figure), the absolute value in Sparrow and Cess's result is seen to be considerably smaller than ours, though a certain degree of estimation has no doubt been made on the effect of the degree of subcooling upon heat transfer. The cause for this is considered to lie in the neglect of the velocity component at the vapor–liquid interface in question, which can be explained also from the fact that the solution expressed by Nu_0 in which there is no degree of subcooling and the velocity component in question is neglected is ranked below the curve Nu^* of Koh's saturated film boiling.

Furthermore, when the degree of superheating is large relative to the degree of subcooling (the right-hand side of the figure), there will arise a contradiction where both $Nu_{sc,A}$ and $Nu_{sc,B}$ have gradually approached to the Nu_0 curve and Koh's value in which no degree of subcooling exists gives a larger heat transfer.

It may be inappropriate to draw general conclusions from this example only, since this problem concerns many parameters, but the authors will provisionally give the following verdict. Sparrow and Cess's solution gives correctly the qualitative effect of the degree of subcooling if it is large, but quantitatively it is rather unsatisfactory. In the case of a small degree of subcooling, their solution can express the effect of the degree of subcooling no longer, since it is obscured by the effect of the neglect of the velocity component in question.

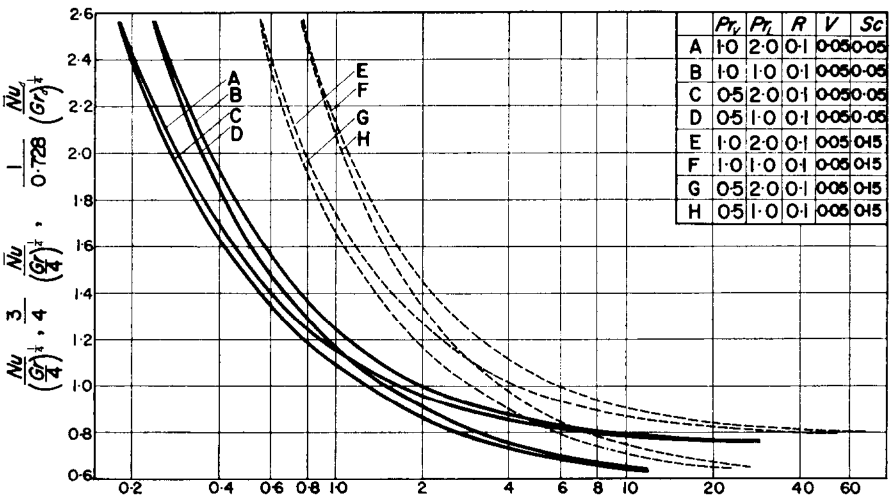
3.3. *Considerations on the results of calculation*

3.3.1. *Effect of Prandtl number.* Figure 6 shows that the Prandtl number of the vapor affects heat transfer intensely with large superheat, and the Prandtl number of the liquid influences it intensely with small superheat. It is quite simple to foresee all this, but the directions of their effects are of interest. That is to say, if the values of the other parameters are fixed, the larger the Prandtl number, the more heat transfer occurs, whereas the larger Prandtl number of the liquid, the less heat transfer. Such a decrease in heat transfer may be ascribed to the circumstances that heat transfer is more strongly affected by hydrodynamic properties than by thermal ones in a liquid and that a large shearing stress occurs at the vapor–liquid interface in the case of a large liquid Prandtl number, checking the upward movement of fluid in the vapor film.

3.3.2. *Effect of degree of subcooling.* As seen in Figs. 6, 8 and 9 the larger the degree of subcooling the more the heat transfer rises, and the increase is greater for a lower degree of superheat. This fact is well-known experimentally.

3.3.3. *Effects of R and V.* As shown in Fig. 7, the larger the $\rho\mu$ ratio and the smaller the dimensionless quantity V, the larger the heat transfer.

Since the fact that the density of the liquid is larger than that of the vapor results in the increase of inertia resistance against the upward



	Pr_v	Pr_l	R	V	Sc
A	1·0	2·0	0·1	0·05	0·05
B	1·0	1·0	0·1	0·05	0·05
C	0·5	2·0	0·1	0·05	0·05
D	0·5	1·0	0·1	0·05	0·05
E	1·0	2·0	0·1	0·05	0·15
F	1·0	1·0	0·1	0·05	0·15
G	0·5	2·0	0·1	0·05	0·15
H	0·5	1·0	0·1	0·05	0·15

$Sp\ \left(=\frac{c_p \Delta T_s}{Pr_v\, l}\right)$

FIG. 6. Effects of Prandtl number and subcooling on heat transfer.

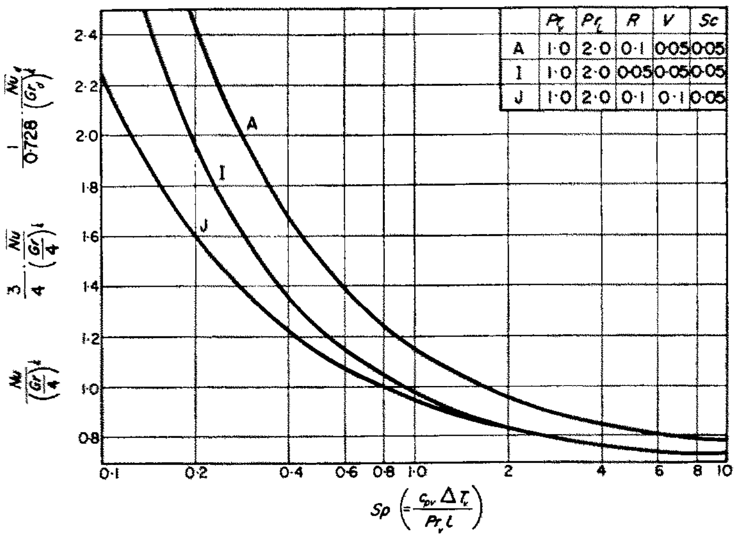


FIG. 7. Effects of R and V on heat transfer.

acceleration of liquid and the fact that the viscosity of the liquid is larger than that of the vapor results in a large viscous resistance against the upward movement of liquid, the liquid is considered, in such a case, to be endowed with properties close to a solid wall, and to have lessened heat transfer. It is easy to understand, in view of the circumstance of such a case resulting from small $\rho\mu$ ratio R, the effect of R upon heat transfer.

With respect to the effect of the dimensionless quantity V, any intuitive interpretation seems to be difficult.

3.3.4. *Result on actual fluid.* Since it is possible in film boiling that the temperature of the heating surface, in the case of water at atmospheric pressure for instance, rises from 100°C up to 1000°C, there remains the question at which temperature the physical properties of vapor should be taken for the calculation of parameters Pr_V, R, V and Sp.

When the relation between dimensionless quantities is read from the results of such an imaginary fluid as shown in Figs. 6 and 7, it will not introduce a serious error to consider the physical properties of the vapor as corresponding to the arithmetic mean of the temperature of the heating surface and that of saturation. When

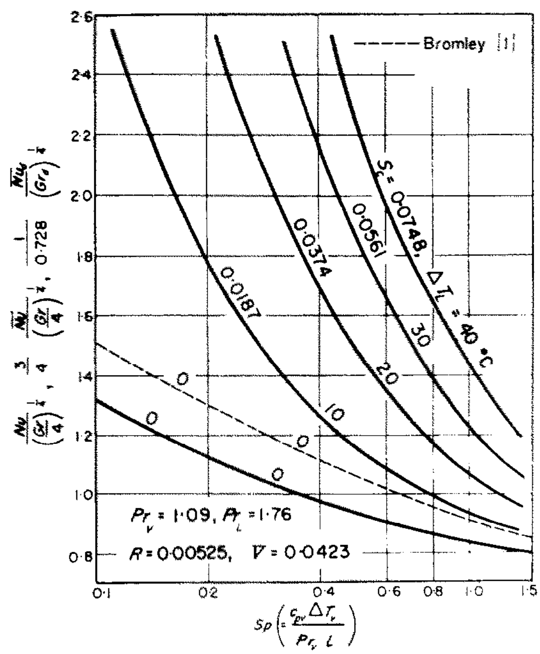


FIG. 8. Non-dimensional representation of heat-transfer characteristics of water at 1 atm.

calculating the characteristics of heat transfer of actual fluids, however, the thickness of the vapor film is designated at the beginning of the calculation, since it is required for the disposition of the analysis, and the degree of superheating is dealt with as an unknown quantity, so that there is no alternative but to give the necessary physical properties of the vapor in the parameters Pr_V, R and V at a provisional temperature.

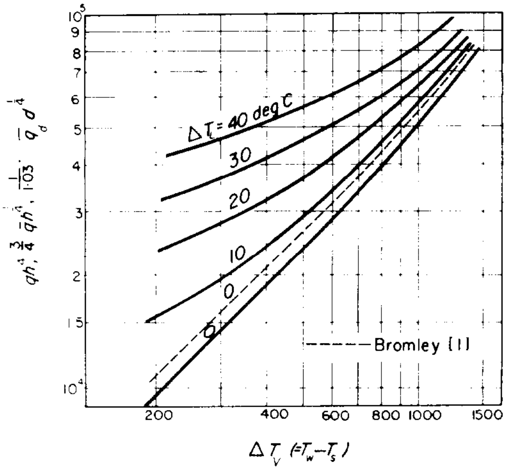


FIG. 9. Boiling curve representation of heat-transfer characteristics of water at 1 atm.

In the case of water under atmospheric pressure calculated here, for a heating surface temperature as high as 1000°C, the values of Pr_V, R and V at the arithmetic mean temperature show no great difference from those for saturation temperature. It is therefore considered that, even if the properties at the mean temperature were adopted only for the calculation of $Nu/(Gr/4)^{\frac{1}{4}}$ and for the degree of dimensionless superheating Sp which are evaluated from the result of calculation and even if values at saturation temperature were used with respect to the parameters Pr_V, R and V in the calculations, the error would be very small. The result thus obtained is shown in Figs. 8 and 9. Incidentally, the curves represented by broken lines are the results given by Bromley's semi-theoretical equation for saturation film boiling, and the solid curves for saturation film boiling are drawn for the values given from Koh's procedures.

4. CONCLUSIONS

By analysing free-convection surface film boiling from a vertical plate by means of the theory of the two-phase boundary layer, and removing the difficulty in the former solution through modification and enlargement and showing its applicability also to the case of a horizontal cylinder, some heat-transfer characteristics of film boiling have been made clear. A prospect has been also opened that differential equations of heat transfer can be integrated even in a parameter domain in which solution by successive approximation procedure converges rather badly, if a large, high speed computer is used.

In the present analysis no consideration has been given to radiative heat transfer, but it is needless to say that, in such a phenomenon as film boiling where the temperature of the heating surface can exceed 1000°C, radiative heat transfer also plays an important part. Putting aside discussions about the synthesis of convective and radiative heat transfer as well as its comparison with measured values, the authors only focus attention here on the latest studies by Sparrow [8]. He analysed a case of heat transfer where convective and radiative heat transfer coexist, using a simple model similar to Bromley's [1], obtaining a result which is said to be in good agreement with the well-known Bromley equation [1] combining convective and radiative heat transfer. This coincidence is remarkable when we come to think of the extremely intuitive deduction with which Bromley has drawn his synthetic equation.

On the other hand, it promises us a favorable turn in the practical field, as it enables us to add radiative heat transfer theoretically to the calculation by means of such a simple relation as Bromley's equation after the analysis of heat transfer for convection only, as shown in this paper.

Now, the difficult point in the application of the analysis mentioned here to an actual problem is that the assumption of the vapor–liquid interface being smooth must be satisfied. In the case of a liquid close to saturation, bubbles are sometimes detached from the vapor–liquid interface or, even though not detached, the interface is wavy, as one of the authors has proved experimentally [9], and its effect has a considerable influence on heat transfer when the degree of subcooling is small. It will, therefore, become necessary to put a certain restriction on the parameter domain when the results mentioned here are applied to actual problems, but even in cases exceeding that limit, the results from the present study may be considered to give standard values from which the fluctuating effect of the vapor–liquid interface is excluded. When the fluctuating effect of the interface or the stirring effect of bubbles has been made known, it will be possible to carry out an analysis on heat transfer also for the case of a liquid close to saturation by combination with the present study.

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APPENDIX

On the Solution of Equation (39)

It is known as Hermann's problem [10] to submit the fundamental differential equations of heat transfer around a horizontal cylinder to a similarity transformation and to find out the functions that transform them into the same differential equations formally as in the case of a vertical plate. In this problem, there exist no ϕ and γ that satisfy the three equations of (39) with appropriate boundary conditions exactly, equation (39) being over conditioned (they are also composed of fundamentally three equations in Hermann's case).

Yamagata induced a solution to satisfy other two equations excepting the second of equation (39) as equation (40). This solution satisfies the second equation of equation (39) only when $X = \pi/2$. Namely, it is virtually an approximate solution of equation (39) in the sense that it satisfies the equation strictly in the neighborhood of $X = \pi/2$, and therefore that it gives approximately a similarity transformation.

Equation (40) is closely related to the function that gives thickness of liquid film when saturated vapor is condensed around a horizontal cylinder referred to in Nusselt's condensation theory. Abramowitz [11] has made a detailed calculation on the following values,

$$I_1 \equiv \int_0^X \sin^{\frac{1}{3}} X \, \mathrm{d}X, \qquad I_2 \equiv \frac{4}{3} \sin^{-\frac{4}{3}} X \int_0^X \sin^{\frac{1}{3}} X \mathrm{d}X.$$

The values of ϕ and γ evaluated by

$$\phi \equiv (4I_1)^{\frac{3}{4}}, \quad \gamma \equiv (3I_2)^{-\frac{1}{4}}$$

are given in the Table A1. The value of γ to be evaluated from the value of this $\bar{\gamma}$ is 0·6122. Incidentally, the value given by Hermann is 0·616.

Table A1. Numerical values of ϕ and γ

X/π	0	1/6	1/3	1/2	2/3	5/6	11/12	1
ϕ	0	1·188	2·343	3·431	4·412	5·235	5·559	5·770
γ	0·7598	0·7494	0·7176	0·6630	0·5812	0·4571	0·3598	0
$1/\gamma$	1·316	1·334	1·394	1·508	1·721	2·188	2·780	∞

Résumé—Une étude théorique du transport de chaleur dans l'ébullition, par film, à partir d'une plaque verticale isotherme, d'un liquide au respos sous-refroidi a été menée en se basant sur la théorie bien connue de la couche limite. Les résultats s'appliquent aussi au transport de chaleur dans l'ébullition par film à partir d'un cylindre horizontal isotherme.

En supposant un film de vapeur mince et stable, les équations de la couche limite ont été transformées en un système d'équations différentielles par des relations de similitude, et l'on trouve que les caractéristiques du transport de chaleur sont corrélées par six paramètres. Notre formulation du problème de l'écoulement diphasique ne diffère de celle de Sparrow et Cess qu'en ce que les conditions de continuité de la vitesse interfaciale et de la contrainte de cisaillement sont requises, au lieu de la condition de vitesse interfaciale nulle employée par eux.

Nous avons d'abord obtenu des solutions pour quelques ensembles arbitrairement choisis de paramètres pour voir comment chaque paramètre affectait le flux de transport de chaleur. Ceci a montré que l'évaluation par Sparrow et Cess des effets de sous-refroidissement sur le coefficient de transport de chaleur est inexacte. Nous avons trouvé alors les caractéristiques de transport de chaleur de l'eau à 1 atm.

Les équations différentielles ont été résolues sur des calculateurs électroniques numériques par approximations successives.

Zusammenfassung—Auf Grund der bekannten Grenzschichttheorie wurde eine theoretische Untersuchung des Wärmeübergangs bei Filmsieden an einer isothermen senkrechten Platte in unterkühlter, ruhender Flüssigkeit durchgeführt. Die Ergebnisse sind auch für den Wärmeübergang bei Filmsieden an einem isothermen waagerechten Zylinder anwendbar. Unter der Annahme eines stabilen Dampffilms wurden die Grenzschichtgleichungen mit Ähnlichkeitsbeziehungen in ein System gewöhnlicher Differentialgleichungen transformiert und die Wärmeübergangscharakteristika liessen sich mit sechs Parametern korrelieren. Die Formulierung des Problems der Zweiphasenströmung in vorliegender Arbeit unterscheidet sich von der Sparrows und Cess nur dadurch, dass hier für die Geschwindigkeit der Zwischenschicht und für die Schubspannung kein Schlupf angenommen wurde während bei ihnen die Geschwindigkeit der Zwischenschicht gleich Null war. Um den Einfluss jedes Parameters auf den Wärmeübergang kennenzulernen, wurden erst Lösungen für beliebige Parametergruppen ermittelt; dabei erwies sich die Auswertung der Unterkühlungswinflüsse auf den Wärmeübergang nach Sparrow und Cess als falsch. Dann fanden wir Wärmeübergangscharakteristika von Wasser bei 1 atm. Differentialgleichungen wurden auf einer elektronischen Digitalrechenmaschine durch fortlaufende Näherung gelöst.

Аннотация—Проведено теоретическое исследование теплообмена при пленочном кипении между изотермической вертикальной пластиной и недогретой стоячей жидкостью. Результаты исследования применимы также для теплообмена при пленочном кипении у изотермического горизонтального цилиндра.

Для стабильной тонкой пленки уравнения пограничного слоя преобразованы ввиду их автомодельности в систему обычных дифференциальных уравнений. Установлено, что теплообменные характеристики описаны с помощью шести параметов. В этой статье формулировка задачи двухфазного течения отличается от формулировки Спэрроу и Цесса тем, что здесь наложены условия отсутствия скольжения скорости на поверхности раздела и наличия напряжения трения, тогда как упомянутые авторы принимали нулевую скорость на поверхности раздела.

Сначала были получены решения для некоторых произвольно выбранных параметров с целью выяснения влияния каждого параметра на скорость теплообмена, что позволило установить ошибочность оценки влияния недогрева на коэффициент теплообмена, сделанной Спэрроу и Цессом. Далее найдены теплообменные характеристики воды при давлении 1 атм.

Дифференциальные уравнения были решены на электронных цифровых вычислительных машинах методом последовательных приближений.